Technical note on converting mass of PM2.5 particles to number of particles

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The formula set out below gives the minimum number of particles in a given particle size range, assuming the particles to be spheres with a diameter equal to that of a sphere the diameter of which is equal to that of the maximum particle size in the size range.

So if the number of particles is N_p then $N_p \geq M_p/m_p$ where M_p is the mass of the particles in that range and m_p is the mass of an individual particle. It also means that: $m_p = (\pi/6)\, d_p^3 \rho_p$ where ρ_p is the particle material density and d_p is the particle diameter.

When the particles are PM2.5 particles then d_p would be 2.5 microns, i.e. the maximum diameter of a PM2.5 particle. It is assumed that the density is that of carbon of 2.25×10^3 kg/m^3. If the emission rate is $\dot{M}_p$ kg/s then the number of particles emitted/s would be: $\dot{N}_p \geq \dot{M}_p/\,(\pi/6)\, d_p^3 \rho_p$. And if $\dot{M}_p$ is expressed as tonnes/year, then you have to convert tonnes/year to kg/s. In this case, where we have $\dot{M}_p$ expressed as tonnes/year, the formula is: $\dot{N}_p = 3.17 \times 10^{-5} \dot{M}_p/\,(\pi/6)\, d_p^3 \rho_p$

Therefore as we know the tonnes of PM2.5 particles released over a year we can estimate the minimum average number of particles of PM2.5 and smaller being released per second, as this is equal to: Tonnes of PM2.5 released over a year × 0.0000317/(2250 × (2.5 × 0.000001)^3 × π/6)

A correct and more general formulaic expression is: $\dot{N}_p = 3.17 \times 10^{-5} \dot{M}_p/\bar{v}_{Dp}\bar{\rho}_p$ [Equation 1] where $\bar{v}_{Dp}$ is the average volume of a particle associated with particles in the range $0 \leq d_p \leq D_p$. This can be written as: $\bar{v}_{Dp} = (\pi/6)\, \bar{d}_p^3$ where $\bar{d}_p$ is the diameter of the equivalent volume sphere of volume $\bar{v}_{Dp}$ and $\bar{\rho}_p$ is the average particle material density.

So the formula in Equation 1 (above) is: $\dot{N}_p = 3.17 \times 10^{-5} \dot{M}_p/(\pi/6)\, D_p^3\, \bar{\rho}_p\, (\bar{d}_p/D_p)^3$

The table below gives values for $(\bar{d}_p/D_p)^3$ assuming a uniform logarithmic spread of the particle diameter. So the value of $\dot{N}_p$ would be the value obtained using the formula in Equation 1 (above) divided by the value given in the table below multiplied by 0.1 to convert the units of measurement.

n	<(dp/Dp)^3>	<vol>
0	1.00E+00	5.24E-01
1	1.45E-01	7.57E-02
2	7.22E-02	3.78E-02
3	4.81E-02	2.52E-02
4	3.60E-02	1.89E-02
5	2.88E-02	1.51E-02
6	2.40E-02	1.26E-02
7	2.05E-02	1.08E-02
8	1.80E-02	9.40E-03

Table 1: Values of $(\bar{d}_p/D_p)^3$ for a uniform logarithmic spread, n refers to the spread, so $n = 1$ means a uniform spread from -1 to 0 logarithmically, i.e. $0.1D_p$ to D_p and n = 2, $0.01D_p$ to D_p etc.